計算の理論 I
ー正則表現とFAの等価性 その2ー

月曜３校時
大月 美佳

連絡事項

- 自転車がまだ散乱してますね

今日の講義内容

1. 前回のミニテスト
   1. ミニテストの解答例
2. 正則表現からDFAまで変換してみる
3. 今日の新しいこと
   1. 正則表現とFAの等価性 つづき
      1. 正則表現からの ε -動作を含むNFAの作り方
         → 例2.12 (p. 42)
      2. DFAからの正則表現の作り方
         → 例2.13 (p. 45)

前回のミニテスト
(演習問題 2.12, p. 66)

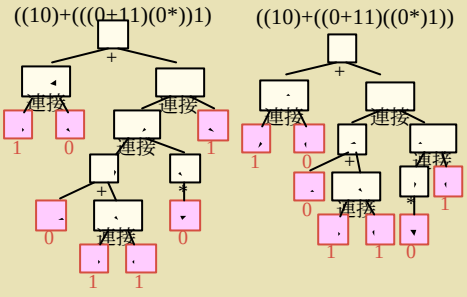
次の正則表現と同値な有限オートマトンを構成せよ。

a) 10+(0+11)0*1

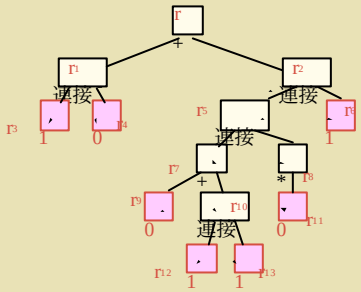
1. 括弧でくくってみる

((10)+(((0+(11))(0*))1) : 左優先
((10)+((0+(11))((0*)1)) : 右優先

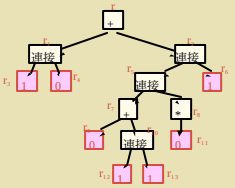
構文木



最初の組合せで考えてみる



式の分解



$r=r_1+r_2$ $r_8=r_{11}^*$

$r_1=r_3r_4$ $r_9=0$

$r_2=r_5r_6$ $r_{10}=r_{12}r_{13}$

$r_3=1$ $r_{11}=0$

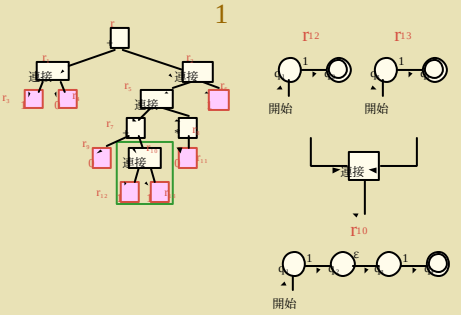
$r_4=0$ $r_{12}=1$

$r_5=r_7r_8$ $r_{13}=1$

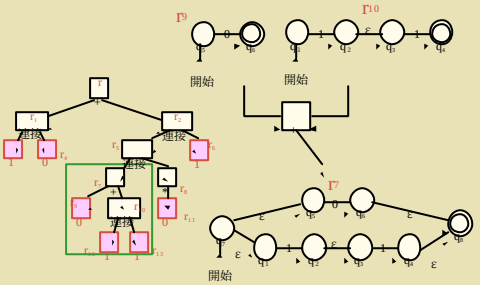
$r_6=1$

$r_7=r_9+r_{10}$

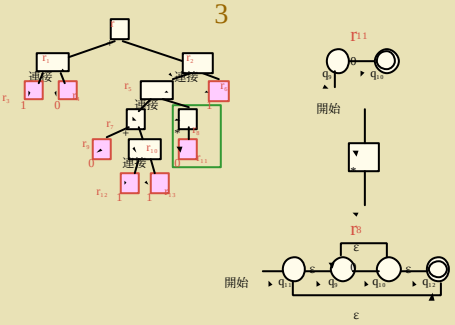
どんどん変換していこう 1



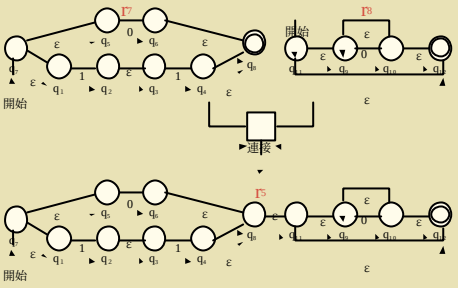
どんどん変換していこう 2



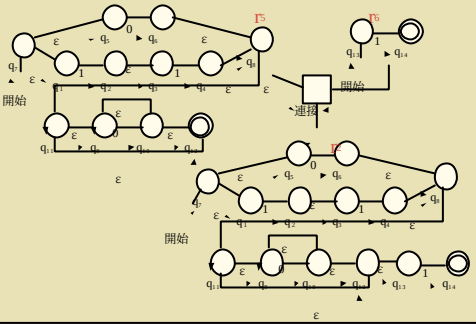
どんどん変換していこう 3



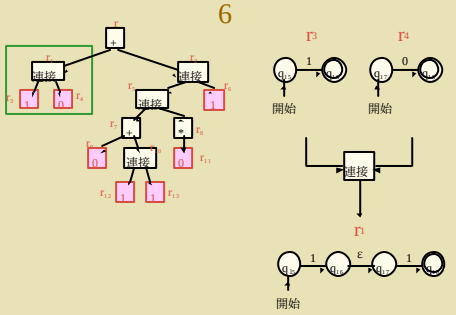
どんどん変換していこう 4



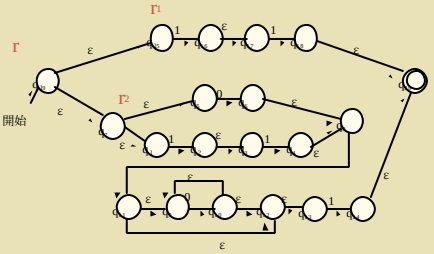
どんどん変換していこう 5



どんどん変換していこう 6



これで（やっと）完成！



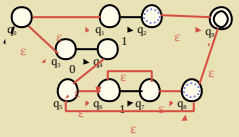
オートマトンとして真面目に書く

- M(Q, {0, 1}, δ, q19, {q20})
- Q={q1, q2, q3, q4, q5, q6, q7, q8, q9, q10, q11, q12, q13, q14, q15, q16, q17, q18, q19, q20}
- δ は右表

δ

	0	1	ε
q0	-	q2	-
q1	-	-	q3
q2	-	q2	-
q3	q4	-	-
q4	-	-	q6
q5	-	-	q6
q6	-	-	q1, q5
q7	-	-	q8
q8	q9	-	-
q9	-	-	q7, q10
q10	-	-	q11
q11	-	q12	-
q12	-	-	q13
q13	-	q14	-
q14	-	-	q15
q15	-	-	q16
q16	-	-	q17, q20
q17	-	q18	-
q18	-	-	q19
q19	-	-	-
q20	-	-	-

前回の例2.12の表記

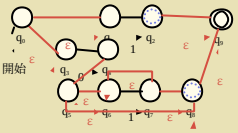


δ

	0	1	ε
q0	-	-	{q1}
q1	-	{q2}	-
q2	-	-	{q3}
q3	{q4}	-	-
q4	-	-	{q6}
q5	-	-	{q6, q1}
q6	{q7}	-	-
q7	-	-	{q8, q10}
q8	-	-	-
q9	-	-	{q7, q10}

- M(Q, {0, 1}, δ, q0, {q9})
- Q={q0, q1, q2, q3, q5, q6, q7, q8, q9}
- δ は右表

ε ありNFAから ε なしNFAを生成してみる



F' ? ? $\begin{cases} F \\ \{q_0\} \end{cases}$? - CLOSURE が F の元を含むとき / そうでないとき

$\hat{?}(q,a)$? ? $\hat{?}(q,a)$ $(q$? Q, a ? ? $)$

$\hat{?}(q_r,a)$? ? $CLOSURE(?(\hat{?}(q_r,?),a))$

$\hat{?}(q_0,?)$? ? ? $CLOSURE(q_i)$

? $\hat{?}(q_r,a)$? ? ? $CLOSURE(?(??CLOSURE(q_i),a))$

- ε -CLOSURE(q_0)={q_0, q_1, q_3}
- ε -CLOSURE(q_1)={q_1}
- ε -CLOSURE(q_2)={q_2, q_3}
- ε -CLOSURE(q_3)={q_3}
- ε -CLOSURE(q_4)={q_4, q_5, q_6, q_7, q_8}
- ε -CLOSURE(q_5)={q_5, q_6, q_7, q_8}
- ε -CLOSURE(q_6)={q_6}
- ε -CLOSURE(q_7)={q_7, q_8, q_9}
- ε -CLOSURE(q_8)={q_8, q_9}
- ε -CLOSURE(q_9)={q_9}

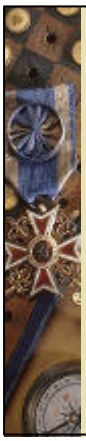
δ ´を求める

q_0について

$?\hat{?}(q_i,a)$? $\hat{?}(q_i,a)$? ? ? $CLOSURE(?(??CLOSURE(q_i),a))$

$\hat{?}(q_0,0)$? ? ? $CLOSURE(?(??CLOSURE(q_0),0))$
? ? ? $CLOSURE(?(\{q_0,q_1,q_3\},0))$
? ? ? $CLOSURE(?(q_0,0)$? $?(q_1,0)$? $?(q_3,0))$
? ? ? $CLOSURE(\bigcirc$? $\bigcirc$? $\{q_4\})$
? ? ? $CLOSURE(q_4)$? $\{q_4,q_5,q_6,q_8,q_9\}$

$\hat{?}(q_0,1)$? ? ? $CLOSURE(?(??CLOSURE(q_0),1))$
? ? ? $CLOSURE(?(\{q_0,q_1,q_3\},1))$
? ? ? $CLOSURE(?(q_0,1)$? $?(q_1,1)$? $?(q_3,1))$
? ? ? $CLOSURE(\bigcirc$? $\{q_2\}$? $\bigcirc)$
? ? ? $CLOSURE(q_2)$? $\{q_2,q_3\}$



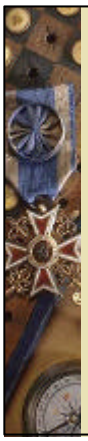
δ´を求める

q_1について

$\hat{\delta}(q_i,a)$? ? $\hat{\delta}(q_i,a)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_i), a))

$\hat{\delta}(q_1,0)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_1), 0))
? ? ? $CLOSURE$ (? ({q_1}, 0))
? ? ? $CLOSURE$ (? (q_1,0))
? ? ? $CLOSURE$ (○)
?○

$\hat{\delta}(q_1,1)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_i), 1))
? ? ? $CLOSURE$ (? ({q_1}, 1))
? ? ? $CLOSURE$ (? (q_1,1))
? ? ? $CLOSURE$ ({q_2})
? ? ? $CLOSURE$ (q_2) ? {q_2, q_9}



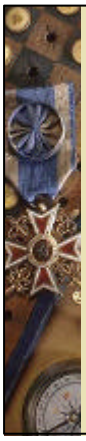
δ´を求める

q_2について

$\hat{\delta}(q_i,a)$? ? $\hat{\delta}(q_i,a)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_i), a))

$\hat{\delta}(q_2,0)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_2), 0))
? ? ? $CLOSURE$ (? ({q_2, q_9}, 0))
? ? ? $CLOSURE$ (? (q_2,0)? ? (q_9,0))
? ? ? $CLOSURE$ (○? ○)
? ? ? $CLOSURE$ (○) ?○

$\hat{\delta}(q_2,1)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_2), 1))
? ? ? $CLOSURE$ (? ({q_2, q_9}, 1))
? ? ? $CLOSURE$ (? (q_2,1) ? ? (q_9,1))
? ? ? $CLOSURE$ (○? ○)
? ? ? $CLOSURE$ (○) ?○



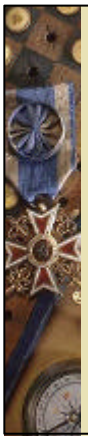
δ´を求める

q_3について

$\hat{\delta}(q_i,a)$? ? $\hat{\delta}(q_i,a)$? ? ? $CLOSURE$(? (? ? $CLOSURE$(q_i), a))

$\hat{\delta}(q_3,0)$? ? ? $CLOSURE$(? (? ? $CLOSURE$(q_3),0)))
? ? ? $CLOSURE$(? ({q_3},0)))
? ? ? $CLOSURE$(? (q_3,0))
? ? ? $CLOSURE$({q_4})
? ? ? $CLOSURE$(q_4) ? {q_4, q_5, q_6, q_8, q_9}

$\hat{\delta}(q_3,1)$? ? ? $CLOSURE$(? (? ? $CLOSURE$(q_3),1))
? ? ? $CLOSURE$(? ({q_3},1)))
? ? ? $CLOSURE$(? (q_3,1))
? ? ? $CLOSURE$(○)
?○



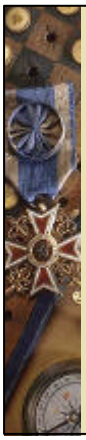
δ´を求める

q_4について

$\hat{\delta}(q_i,a)$? ? $\hat{\delta}(q_i,a)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_i), a))

$\hat{\delta}(q_4,0)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_4), 0))
? ? ? $CLOSURE$ (? ({q_4, q_5, q_6, q_8, q_9},0))
? ? ? $CLOSURE$ (? (q_4,0)? ? (q_5,0)? ? (q_6,0)? ? (q_8,0)? ? (q_9,0))
? ? ? $CLOSURE$ (○? ○? ○? ○? ○)
? ? ? $CLOSURE$ (○) ?○

$\hat{\delta}(q_4,1)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_4), 1))
? ? ? $CLOSURE$ (? ({q_4, q_5, q_6, q_8, q_9},1))
? ? ? $CLOSURE$ (? (q_4,1)? ? (q_5,1) ? ? (q_6,1) ? ? (q_8,1)? ? (q_9,1))
? ? ? $CLOSURE$ (○? ○? {q_7} ? ○? ○)
? ? ? $CLOSURE$ (q_7) ? {q_7, q_6, q_8, q_9}



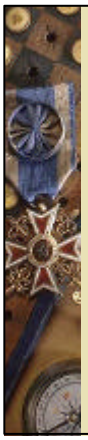
δ´を求める

q_5について

$\hat{\delta}(q_i,a)$? ? $\hat{\delta}(q_i,a)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_i), a))

$\hat{\delta}(q_5,0)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_5),0))
? ? ? $CLOSURE$ (? ({q_5, q_6, q_8, q_9},0))
? ? ? $CLOSURE$ (? (q_5,0) ? ? (q_6,0) ? ? (q_8,0) ? ? (q_9,0))
? ? ? $CLOSURE$ (○? ○? ○? ○)
? ? ? $CLOSURE$ (○) ?○

$\hat{\delta}(q_5,1)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_5),1))
? ? ? $CLOSURE$ (? ({q_5, q_6, q_8, q_9},1))
? ? ? $CLOSURE$ (? (q_5,1) ? ? (q_6,1) ? ? (q_8,1) ? ? (q_9,1))
? ? ? $CLOSURE$ (○? {q_7} ? ○? ○)
? ? ? $CLOSURE$ (q_7) ? {q_6, q_7, q_8, q_9}



δ´を求める

q_6について

$\hat{\delta}(q_i,a)$? ? $\hat{\delta}(q_i,a)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_i), a))

$\hat{\delta}(q_6,0)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_6),0))
? ? ? $CLOSURE$ (? ({q_6},0))
? ? ? $CLOSURE$ (? (q_6,0))
? ? ? $CLOSURE$ (○)
?○

$\hat{\delta}(q_6,1)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_6),1))
? ? ? $CLOSURE$ (? ({q_6},1))
? ? ? $CLOSURE$ (? (q_6,1))
? ? ? $CLOSURE$ (q_7)
? {q_6, q_7, q_8, q_9}

δ´を求める
q_7について

$\hat{\delta}'(q_i,a)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_i), a))

$\hat{\delta}'(q_7,0)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_7),0))
? ? ? $CLOSURE$ (? ({q_6, q_7, q_8, q_9},0))
? ? ? $CLOSURE$ (? (q_6,0)? ? (q_7,0)? ? (q_8,0) ? ? (q_9,0))
? ? ? $CLOSURE$ (○? ○? ○? ○)
? ? ? $CLOSURE$ (○) ?○

$\hat{\delta}'(q_7,1)$? ? ? $CLOSURE$ (? (? $CLOSURE$ (q_7),1))
? ? ? $CLOSURE$ (? ({q_6, q_7, q_8, q_9},1))
? ? ? $CLOSURE$ (? (q_6,1) ? ? (q_7,1) ? ? (q_8,1)? ? (q_9,1))
? ? ? $CLOSURE$ ({q_7}? ○? ○? ○)
? ? ? $CLOSURE$ (q_7) ? {q_6, q_7, q_8, q_9}

δ´を求める
q_8について

$\hat{\delta}'(q_i,a)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_i), a))

$\hat{\delta}'(q_8,0)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_8),0))
? ? ? $CLOSURE$ (? ({q_8, q_9},0))
? ? ? $CLOSURE$ (? (q_8,0) ? ? (q_9,0))
? ? ? $CLOSURE$ (○? ○)
? ? ? $CLOSURE$ (○) ?○

$\hat{\delta}'(q_8,1)$? ? ? $CLOSURE$ (? (? ? $CLOSURE$ (q_8),1))
? ? ? $CLOSURE$ (? ({q_8, q_9},1))
? ? ? $CLOSURE$ (? (q_8,1) ? ? (q_9,1))
? ? ? $CLOSURE$ (○? ○)
? ? ? $CLOSURE$ (○) ?○

δ´を求める
q_9について

$\hat{\delta}'(q_i,a)$? ? ? $CLOSURE$(? (? $CLOSURE$(q_i),a))

$\hat{\delta}'(q_9,0)$? ? ? $CLOSURE$(? (? ? $CLOSURE$(q_9),0))
? ? ? $CLOSURE$(? ({q_9},0))
? ? ? $CLOSURE$(? (q_9,0))
? ? ? $CLOSURE$(○)
?○

$\hat{\delta}'(q_9,1)$? ? ? $CLOSURE$(? (? ? $CLOSURE$(q_9),1))
? ? ? $CLOSURE$(? ({q_9},1))
? ? ? $CLOSURE$(? (q_9,1))
? ? ? $CLOSURE$(○)
?○

生成されたNFA

δ´

	0	1
q_0	{q_0, q_1, q_2, q_3, q_4}	{q_5, q_8}
q_1	-	{q_5, q_8}
q_2	-	-
q_3	{q_3, q_4, q_6, q_7, q_9}	-
q_4	-	{q_4, q_7, q_8, q_9}
q_5	-	{q_8, q_7, q_8, q_9}
q_6	-	{q_4, q_7, q_8, q_9}
q_7	-	{q_8, q_7, q_8, q_9}
q_8	-	-
q_9	-	-

M´(Q´, {0,1}, δ´,q_0,{q_9})
Q´={q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8, q_9}
δ´は右表

さらにDFAへ変換してみる

δ´

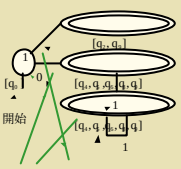
	0	1
q_0	{q_0,q_1,q_2,q_3,q_4}	{q_5,q_8}
q_1	-	{q_5,q_8}
q_2	-	-
q_3	{q_3,q_4,q_6,q_7,q_9}	-
q_4	-	{q_4,q_7,q_8,q_9}
q_5	-	{q_8,q_7,q_8,q_9}
q_6	-	{q_4,q_7,q_8,q_9}
q_7	-	{q_8,q_7,q_8,q_9}
q_8	-	-
q_9	-	-

δ´´

	0	1
[q_0]	[q_0,q_1,q_2,q_3,q_4]	[q_5,q_8]
[q_1,q_2,q_3,q_4]	-	[q_5,q_8,q_7,q_9]
[q_5,q_8]	-	[q_8,q_7,q_8,q_9]
[q_8,q_7,q_9]	-	[q_8,q_7,q_8,q_9]

M´´(Q´´, {0,1}, δ´´,[q_0],F)
Q´´={[q_0], [q_4, q_5, q_6, q_8, q_9], [q_2, q_9], [q_6, q_7, q_8, q_9]}
δ´´は上表
F= {[q_4, q_5, q_6, q_8, q_9], [q_2, q_9], [q_6, q_7, q_8, q_9]}

もとの正則表現と比べてみる



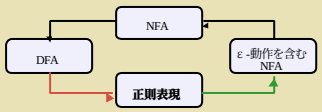
δ´´

	0	1
[q_0]	[q_0,q_1,q_2,q_3,q_4]	[q_5,q_8]
[q_1,q_2,q_3,q_4]	-	[q_5,q_8,q_7,q_9]
[q_5,q_8]	-	[q_8,q_7,q_8,q_9]
[q_8,q_7,q_9]	-	[q_8,q_7,q_8,q_9]

01*+1

今日の新しいこと

1. 正則表現とFAの等価性



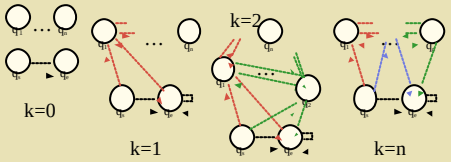
1. 正則表現からの ε -動作を含むNFAの作り方
   →NFA の生成例 (例2.12, p. 42)　先週やった
2. DFAからの正則表現の作り方
   →正則表現の生成例 (例2.13, p. 45)

DFAからの正則表現の作り方

考え方

1. ある状態からある状態の間の状態を0からひとつずつ増やしていって、
2. 途中の状態の任意の組からなる道が生成することのできる文字列の正則表現を再帰的に拡張していき、
3. 最後に、初期状態から最終状態への道が生成できる文字列の正則表現を求める。

状態を増やしていくとは？



$$R_{ij}^{k} = R_{ik}^{k-1}(R_{kk}^{k-1})^{*}R_{kj}^{k-1} \cup R_{ij}^{k-1} \quad (k \geq 1)$$

$$R_{ij}^{0} = \{a \mid \delta(q_i, a) = q_j\} \cup \{\varepsilon\} \quad (i = j \text{のとき})$$

$$R_{ij}^{0} = \{a \mid \delta(q_i, a) = q_j\} \quad (i \neq j \text{のとき})$$

正則表現の式との対応

$$R_{ij}^{k} = R_{ik}^{k-1}(R_{kk}^{k-1})^{*}R_{kj}^{k-1} \cup R_{ij}^{k-1} \quad (k \geq 1)$$

$$R_{ij}^{0} = \{a \mid \delta(q_i, a) = q_j\} \cup \{\varepsilon\} \quad (i = j \text{のとき})$$

$$R_{ij}^{0} = \{a \mid \delta(q_i, a) = q_j\} \quad (i \neq j \text{のとき})$$

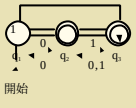
↓ 証明はp.44～45

$$r_{ij}^{k} = r_{ik}^{k-1}(r_{kk}^{k-1})^{*}r_{kj}^{k-1} \cup r_{ij}^{k-1} \quad (k \geq 1)$$

$$r_{ij}^{0} = a \cup \varepsilon \quad (i = j \text{のとき})$$

$$r_{ij}^{0} = a \quad (i \neq j \text{のとき})$$

正則表現の生成例
(例2.13, p. 45)

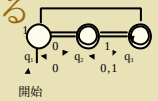


δ		
	0	1
q_1	q_2	q_3
q_2	q_1	q_3
q_3	q_2	q_2

$r_{11}^{0} = \{a \mid \delta(q_1,a) = q_1\} \cup \{\varepsilon\}$　(1, 1)
$r_{12}^{0} = \{a \mid \delta(q_1,a) = q_2\}$　0　(1, 2)
$r_{13}^{0} = \{a \mid \delta(q_1,a) = q_3\}$　1　(1, 3)
$r_{21}^{0} = \{a \mid \delta(q_2,a) = q_1\}$　0　(2, 1)
$r_{22}^{0} = \{a \mid \delta(q_2,a) = q_2\} \cup \{\varepsilon\}$　(2, 2)
$r_{23}^{0} = \{a \mid \delta(q_2,a) = q_3\}$　1　(2, 3)
$r_{31}^{0} = \{a \mid \delta(q_3,a) = q_1\}$　∅　(3, 1)
$r_{32}^{0} = \{a \mid \delta(q_3,a) = q_2\}$　0 ∪ 1　(3, 2)
$r_{33}^{0} = \{a \mid \delta(q_3,a) = q_3\} \cup \{\varepsilon\}$　(3, 3)

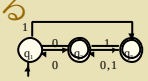
どんどん進める

k=1



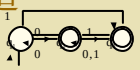
$r_{11}^{1} = r_{11}^{0}(r_{11}^{0})^{*}r_{11}^{0} \cup r_{11}^{0} = \varepsilon(\varepsilon)^{*}\varepsilon \cup \varepsilon = \varepsilon$
$r_{12}^{1} = r_{11}^{0}(r_{11}^{0})^{*}r_{12}^{0} \cup r_{12}^{0} = \varepsilon(\varepsilon)^{*}0 \cup 0 = 0 \cup 0 = 0$
$r_{13}^{1} = r_{11}^{0}(r_{11}^{0})^{*}r_{13}^{0} \cup r_{13}^{0} = \varepsilon(\varepsilon)^{*}1 \cup 1 = 1 \cup 1 = 1$
$r_{21}^{1} = r_{21}^{0}(r_{11}^{0})^{*}r_{11}^{0} \cup r_{21}^{0} = 0(\varepsilon)^{*}\varepsilon \cup 0 = 0 \cup 0 = 0$
$r_{22}^{1} = r_{21}^{0}(r_{11}^{0})^{*}r_{12}^{0} \cup r_{22}^{0} = 0(\varepsilon)^{*}0 \cup \varepsilon = 00 \cup \varepsilon$
$r_{23}^{1} = r_{21}^{0}(r_{11}^{0})^{*}r_{13}^{0} \cup r_{23}^{0} = 0(\varepsilon)^{*}1 \cup 1 = 01 \cup 1$
$r_{31}^{1} = r_{31}^{0}(r_{11}^{0})^{*}r_{11}^{0} \cup r_{31}^{0} = \emptyset(\varepsilon)^{*}\varepsilon \cup \emptyset = \emptyset \cup \emptyset$
$r_{32}^{1} = r_{31}^{0}(r_{11}^{0})^{*}r_{12}^{0} \cup r_{32}^{0} = \emptyset(\varepsilon)^{*}0 \cup 0 = \emptyset \cup 1 = 0 \cup 1$
$r_{33}^{1} = r_{31}^{0}(r_{11}^{0})^{*}r_{13}^{0} \cup r_{33}^{0} = \emptyset(\varepsilon)^{*}1 \cup \varepsilon = \emptyset \cup \varepsilon$

どんどん進める
k=2



$r_{11}^2 ? r_{12}^1 (r_{22}^1 *) r_{21}^1 ? r_{11}^1 ? 0((00? ?)*)0? ? ? 0(00)*0 ? ? ? (00)*$
$r_{12}^2 ? r_{12}^1 (r_{22}^1 *) r_{22}^1 ? r_{12}^1 ? 0((00? ?)*)(00? ?) ? 0 ? 0(00)*?0 ? 0(00)*$
$r_{13}^2 ? r_{12}^1 (r_{22}^1 *) r_{23}^1 ? r_{13}^1 ? 0((00? ?)*)(01? 1) ? 1 ? 0(00)*(0? ?)1 ? 1$
$? 00*1? 1 ? 0*1$
$r_{21}^2 ? r_{22}^1 (r_{22}^1 *) r_{21}^1 ? r_{21}^1 ? (00? ?)((00? ?)*)0 ? 0 ? (00)*0? 0 ? (00)*0$
$r_{22}^2 ? r_{22}^1 (r_{22}^1 *) r_{22}^1 ? r_{22}^1 ? (00? ?)((00? ?)*)(00? ?) ? (00? ?)$
$? (00)*?00? ? ? (00)*$
$r_{23}^2 ? r_{22}^1 (r_{22}^1 *) r_{23}^1 ? r_{23}^1 ? (00? ?)((00? ?)*)(01? 1) ? (01? 1)$
$? (00)*(0? ?)1 ? 01? 1 ? 0*1? 0*1? 1 ? 0*1$
$r_{31}^2 ? r_{32}^1 (r_{22}^1 *) r_{21}^1 ? r_{31}^1 ? (0? 1)((00? ?)*)0? ○? (0? 1)(00)*0$
$r_{32}^2 ? r_{32}^1 (r_{22}^1 *) r_{22}^1 ? r_{32}^1 ? (0? 1)((00? ?)*)(00? ?) ? (0 ? 1)$
$? (0 ? 1)(00)*?(0? 1) ? (0? 1)(00)*$
$r_{33}^2 ? r_{32}^1 (r_{22}^1 *) r_{23}^1 ? r_{33}^1 ? (0? 1)((00? ?)*)(01? 1) ? ?$
$? (0 ? 1)(00)*(01? 1) ? ? ? (0 ? 1)(00)*(0? ?)1 ? ? ? (0? 1)0*1? ?$

最終状態への道
k=3



$r_{12}^3 ? r_{13}^2 (r_{33}^2 *) r_{32}^2 ? r_{12}^2$
$? (0*1)(((0? 1)0*1 ? ?)*)(0 ? 1)(00)*?0(00)*$
$? 0*1((0? 1)0*1)*(0? 1)(00)*?0(00)*$

$r_{13}^3 ? r_{13}^2 (r_{33}^2 *) r_{33}^2 ? r_{13}^2$
$? 0*1(((0? 1)0*1 ? ?)*)((0 ? 1)0*1? ?) ? (0*1)$
$? 0*1((0? 1)0*1)*((0 ? 1)0*1 ? ?) ? 0*1$
$? 0*1((0? 1)0*1)*$

$r_{12}^3 ? r_{13}^3 ? 0*1((0 ? 1)0*1)*(0 ? 1)(00)*?(00)*?0*1((0 ? 1)0*1)*$
$? 0*1((0? 1)0*1)*((0 ? 1)(00)*??) ? 0(00)*$

今日のミニテスト

- ミニテスト
  - 演習問題 2.13のa
  - 教科書・資料を見ても良い
- 資料、ミニテストがない人は前へ
- 提出したら帰って良し
- 次回
  - 3章いきます。反復補題。