計算の理論 I
ε-動作を含むNFA

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今日の講義内容

1. ε-動作を含むNFA遷移関数と受理
   復習・補足
2. ε-動作ありNFA→ε-動作なしNFA
3. ε-動作なしNFA→DFA
4. ミニテスト

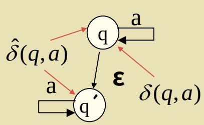
$\hat{\delta}$の定義

1) $\hat{\delta}(q,\varepsilon)=\varepsilon-CLOSURE(q)$

2) $\hat{\delta}(q,wa)=\varepsilon-CLOSURE(P)$

ただし、$w\in\Sigma^*, q\in\Sigma,$

$P=\{p\,|$ ある$\hat{\delta}(q,w)$の元rに対して$p\in\delta(r,a)\}$



$\boxed{\hat{\delta}(q,a)}$は必ずしも$\boxed{\delta(q,a)}$と等しくない。

qからaの辺で直接到達できる状態の集合

qからaをラベルに持つ道（εを含む）を通って到達できる状態の集合

δと$\hat{\delta}$の拡張

1) $\hat{\delta}(q,\varepsilon)=\varepsilon-CLOSURE(q)$

2) $\hat{\delta}(q,wa)=\varepsilon-CLOSURE(P)$

ただし、$w\in\Sigma^*, q\in\Sigma,$

$P=\{p\,|$ ある$\hat{\delta}(q,w)$の元rに対して$r\in\delta(r,a)\}$

さらに、状態の集合$R(\subseteq Q)$に対して

3) $\delta(\mathrm{R},a)=\bigcup_{q\in R}\delta(q,a)$

4) $\hat{\delta}(\mathrm{R},w)=\bigcup_{q\in R}\hat{\delta}(q,w)$

ε-動作ありNFAの受理言語

◆ **定義**

$M=(Q,\Sigma,\delta,q_0,F)$が受理する言語は

$\{w \mid \hat{\delta}(q_0,w)$はFの元を含む$\}$

であり$L(M)$と書く。

受理の例

1. 0: $\hat{\delta}(q_0,0)=\varepsilon-CLOSURE(\delta(\hat{\delta}(q_0,\varepsilon),0))$

$$=\varepsilon-CLOSURE(\delta(\{q_0,q_1,q_2\},0))$$
$$=\varepsilon-CLOSURE(\delta(q_0,0)\cup\delta(q_1,0)\cup\delta(q_2,0))$$
$$=\varepsilon-CLOSURE(\{q_0\}\cup\phi\cup\phi)$$
$$=\varepsilon-CLOSURE(\{q_0\})=\{q_0,q_1,q_2\}$$

2. 01: $\hat{\delta}(q_0,01)=\varepsilon-CLOSURE(\delta(\hat{\delta}(q_0,0),1))$

$$=\varepsilon-CLOSURE(\delta(\{q_0,q_1,q_2\},1))$$
$$=\varepsilon-CLOSURE(\delta(q_0,1)\cup\delta(q_1,1)\cup\delta(q_2,1))$$
$$=\varepsilon-CLOSURE(\phi\cup\{q_1\}\cup\phi)$$
$$=\varepsilon-CLOSURE(\{q_1\})=\{q_1,q_2\}$$

ε-動作なしNFAとε-動作ありNFAの等価性

ε-動作ありNFA: M (Q, Σ, δ, q0, F)が
ε-動作なしNFA: M′(Q, Σ, δ′, q0, F′)
で模倣できる（帰納法）。

$$F'=\begin{cases}F\cup\{q_0\} & \varepsilon\text{-CLOSUREがFの元を含むとき}\\ F & \text{そうでないとき}\end{cases}$$

$$\delta'(q,a)=\hat{\delta}(q,a)\quad(q\in Q, a\in\Sigma)$$

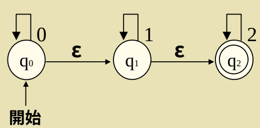
ε-動作ありNFAを模倣するε-動作なしNFAの例

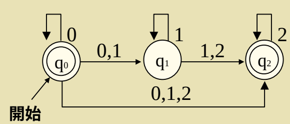
δ

	0	1	2	ε
q_0	$\{q_0\}$	φ	φ	$\{q_1\}$
q_1	φ	$\{q_1\}$	φ	$\{q_2\}$
q_2	φ	φ	$\{q_2\}$	φ

δ′

	0	1	2
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	φ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	φ	φ	$\{q_2\}$



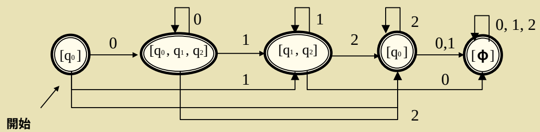


ε-遷移無しNFAからDFAへ

サブセット構成法

	0	1	2
$*\to q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
$*q_2$	ϕ	ϕ	$\{q_2\}$

	0	1	2
$*\to[q_0]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$	$[q_2]$
$*[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$	$[q_1, q_2]$	$[q_2]$
$*[q_1, q_2]$	$[\phi]$	$[q_1, q_2]$	$[q_2]$
$*[q_2]$	$[\phi]$	$[\phi]$	$[q_2]$
$[\phi]$	$[\phi]$	$[\phi]$	$[\phi]$



ε-動作を含むNFA 例1

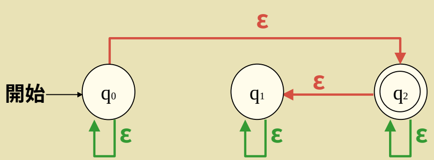
- ε-動作を含むNFA

$(\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_2\})$

δは下表

	0	1	ε
q_0	q_0, q_1	-	q_2
q_1	q_2	q_0, q_1	-
q_2	q_0, q_1	-	q_1

例1 ε-CLOSURE



	0	1	ε
q_0	q_0, q_1	-	q_2
q_1	q_2	q_0, q_1	-
q_2	q_0, q_1	-	q_1

q	ECLOSE(q)
q_0	q_0, q_1, q_2
q_1	q_1
q_2	q_1, q_2

例1 NFA

$\delta'(q_0, 0)=\mathrm{EClOSE}(\delta(\mathrm{ECLOSE}(q_0), 0)) = \mathrm{EClOSE}(\delta(\{q_0, q_1, q_2\}, 0))$
$= \mathrm{EClOSE}(\{q_0, q_1, q_2\})= \{q_0, q_1, q_2\}$

$\delta'(q_0, 1)=\mathrm{EClOSE}(\delta(\mathrm{ECLOSE}(q_0), 1)) = \mathrm{EClOSE}(\delta(\{q_0, q_1, q_2\}, 1))$
$= \mathrm{EClOSE}(\{q_0, q_1\})= \{q_0, q_1, q_2\}$

$\delta'(q_1, 0)=\mathrm{EClOSE}(\delta(\mathrm{ECLOSE}(q_1), 0)) = \mathrm{EClOSE}(\delta(\{q_1\}, 0))$
$=\mathrm{EClOSE}(\{q_2\})=\{q_1, q_2\}$

$\delta'(q_1, 1)=\mathrm{EClOSE}(\delta(\mathrm{ECLOSE}(q_1), 1)) = \mathrm{EClOSE}(\delta(\{q_1\}, 1))$
$= \mathrm{EClOSE}(\delta(\{q_0, q_1\}, 1))=\{q_0, q_1, q_2\}$

$\delta'(q_2, 0)=\mathrm{EClOSE}(\delta(\mathrm{ECLOSE}(q_2), 0)) = \mathrm{EClOSE}(\delta(\{q_1, q_2\}, 0))$
$= \mathrm{EClOSE}(\{q_0, q_1, q_2\}) = \{q_0, q_1, q_2\}$

$\delta'(q_2, 1)=\mathrm{EClOSE}(\delta(\mathrm{ECLOSE}(q_2), 1)) = \mathrm{EClOSE}(\delta(\{q_1, q_2\}, 1))$
$= \mathrm{EClOSE}(\{q_0, q_1\}) = \{q_0, q_1, q_2\}$

	0	1
$*\to q_0$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$
q_1	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
$*q_2$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$

例1 NFA→DFA

サブセット構成法

	0	1
*→q_0	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$
q_1	$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$
*q_2	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$

	0	1
*→$[q_0]$	$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$
*$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$

ε-動作を含むNFA→NFA 例2

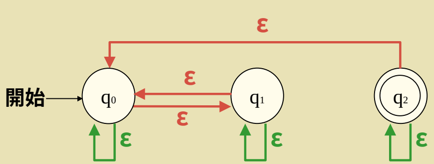
- ε-動作を含むNFA

$(\{q_0, q_1, q_2\}, \{0, 1\}, \delta, q_0, \{q_2\})$

δは下表

	0	1	ε
q_0	q_0	-	q_1
q_1	-	q_0, q_2	q_0
q_2	q_1	q_2	q_0

例2 ε-CLOSURE



	0	1	ε
q_0	q_0	-	q_1
q_1	-	q_0, q_2	q_0
q_2	q_1	q_2	q_0

q	ECLOSE
q_0	q_0, q_1
q_1	q_0, q_1
q_2	q_0, q_1, q_2

例2 NFA

$\delta'(q_0, 0)$=EClOSE(δ(EClOSE(q_0), 0))=

$\delta'(q_0, 1)$=EClOSE(δ(EClOSE(q_0), 1))=

$\delta'(q_1, 0)$=EClOSE(δ(EClOSE(q_1), 0))=

$\delta'(q_1, 1)$=EClOSE(δ(EClOSE(q_1), 1))=

$\delta'(q_2, 0)$=EClOSE(δ(EClOSE(q_2), 0))=

$\delta'(q_2, 1)$=EClOSE(δ(EClOSE(q_2), 1))=

	0	1
→q_0		
q_1		
*q_2		

例2 NFA→DFA

サブセット構成法

	0	1
→q_0		
q_1		
*q_2		

	0	1
→[q_0]		

ミニテスト と次回内容

- **ミニテスト**
  教科書･資料を見ても、友達と相談しても良い
  15**分後に指名された人は板書**
- **ミニテストを提出すること**
  出したら帰って良し
- **次回**(6/9)**内容**
  正則（正規）表現